

Rational functions

What is a rational function?

A function of the form: $f(x) = \frac{p(x)}{q(x)}$

← Polynomial

← Polynomial not equal to 0

The inverse variation function is a rational function

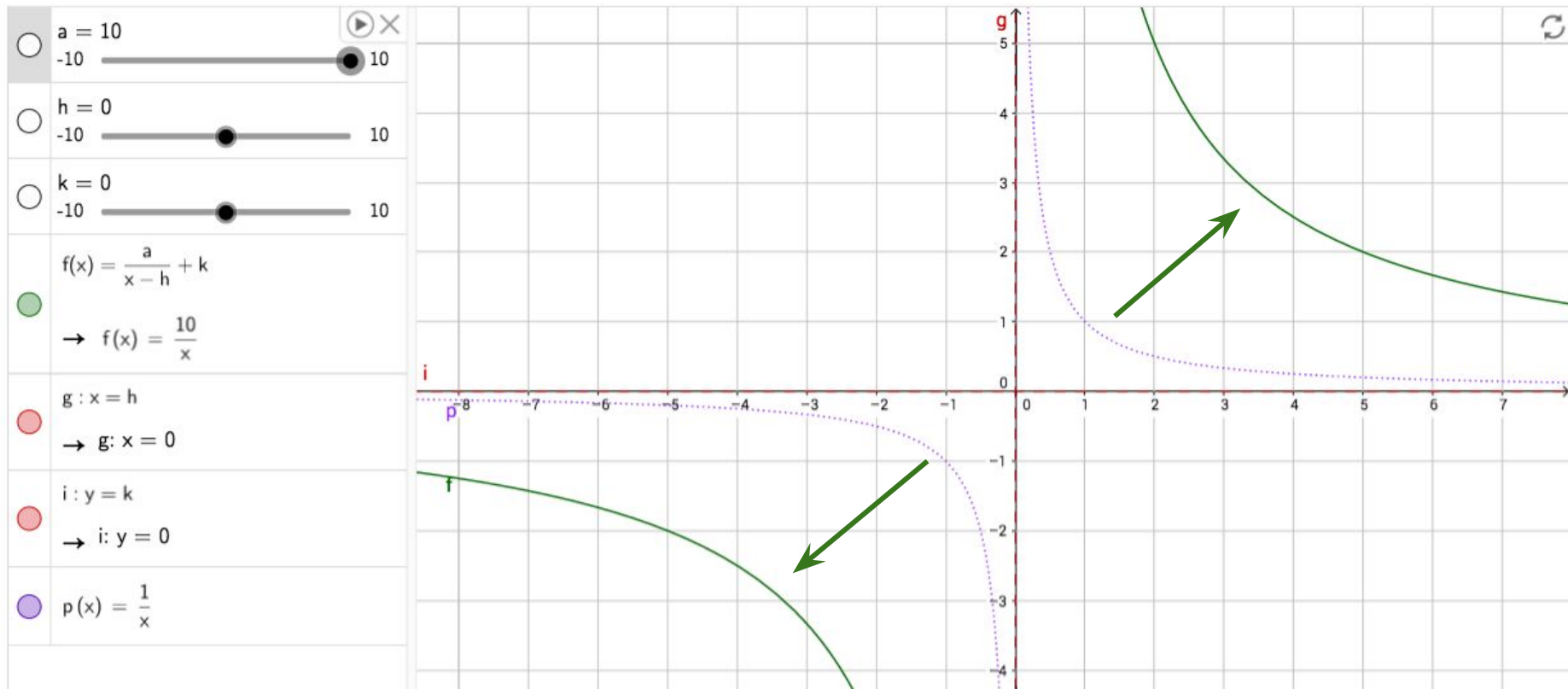
Graphs

Parent function

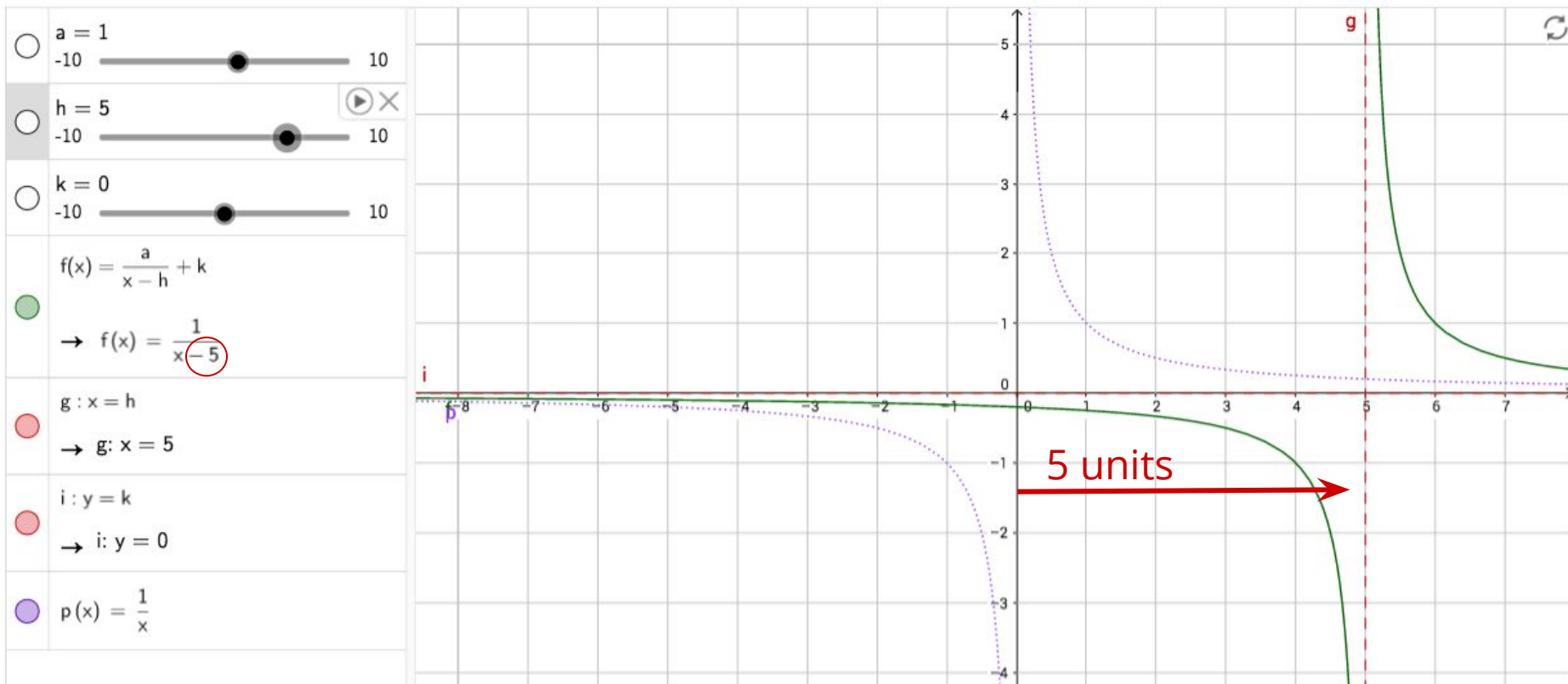
Slider on a

Slider on h and k

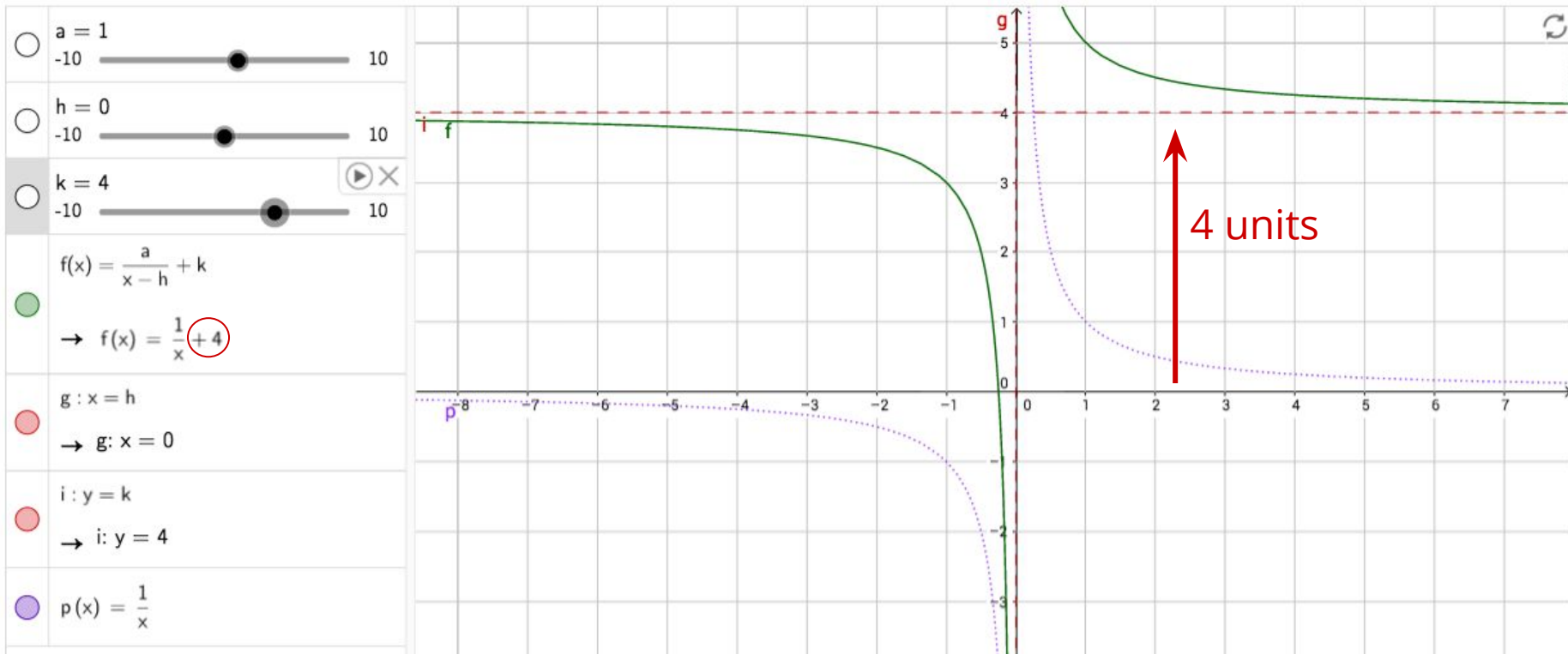
As a gets further away from 1 or -1, the branches of the *hyperbola* move further away from the asymptotes.





As h decreases, the asymptotes shift in the positive direction the same number.




As k increases, the asymptotes shift in the positive direction the same number.



$a = 10$
 -10  10

$h = 5$
 -10  10

$k = 4$
 -10  10

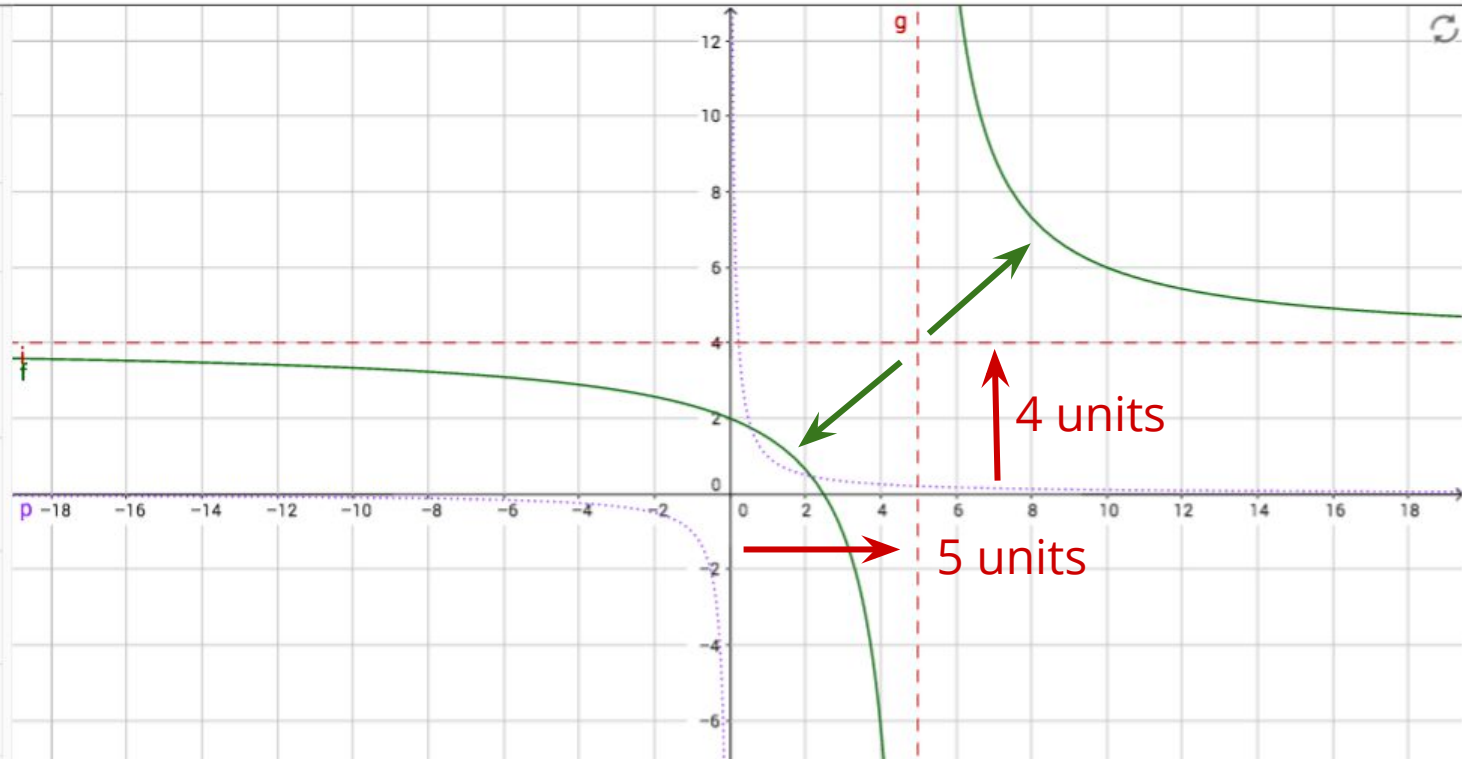
$f(x) = \frac{a}{x-h} + k$

$\rightarrow f(x) = \frac{10}{x-5} + 4$

$g: x = h$
 $\rightarrow g: x = 5$

$i: y = k$
 $\rightarrow i: y = 4$

$p(x) = \frac{1}{x}$



Sketch the graph of the functions:

$$f(x) = \frac{4}{x} + 2$$

$$f(x) = \frac{3}{x+1} - 2$$

$$f(x) = -\frac{4}{x}$$

$$f(x) = \frac{4}{x-1} + 1$$

This is also a simple rational function: $f(x) = \frac{ax + b}{cx + d}$

It's asymptotes are at: $y = \frac{a}{c}$ and $x = -\frac{d}{c}$

Simple Rational Function

Sketch the graph of the functions:

$$f(x) = \frac{2x + 1}{x - 3}$$

$$f(x) = \frac{5x + 3}{-x + 10}$$

$$f(x) = \frac{6x - 1}{3x - 1}$$

A long-distance calling plan has a fixed monthly fee of \$4.95 and costs 5 cents a minute.

Write an equation that gives your average cost C (in dollars) per minute m during a given month.

Estimate when the average cost is \$0.14 per minute.

[Graph](#)

An internet service provider charges a \$50 installation fee and a monthly fee of \$43.

Write and graph an equation that gives the average cost per month as a function of the number of months of service.

Estimate after how many months will the average cost be \$53?