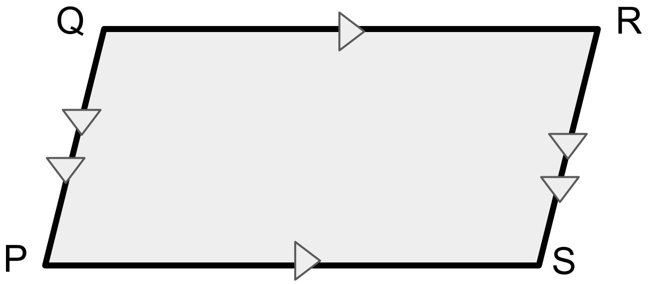
Properties of Parallelograms

What is a parallelogram?

A quadrilateral with both pairs of opposite sides parallel.

Notation: ▱PQRS

$\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$



Theorem 8.3

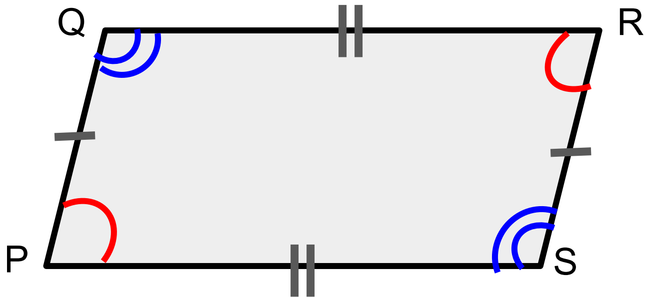
If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If PQRS is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$

Theorem 8.4

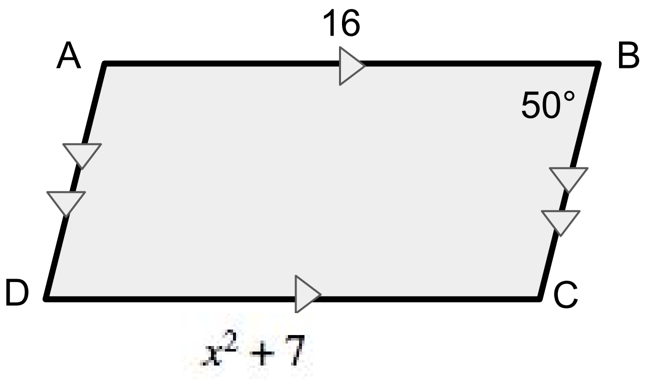
If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If PQRS is a parallelogram, then $\angle P \cong \angle R$ and $\angle \cong \angle S$



ABCD is a parallelogram,

Find the missing values:



x=

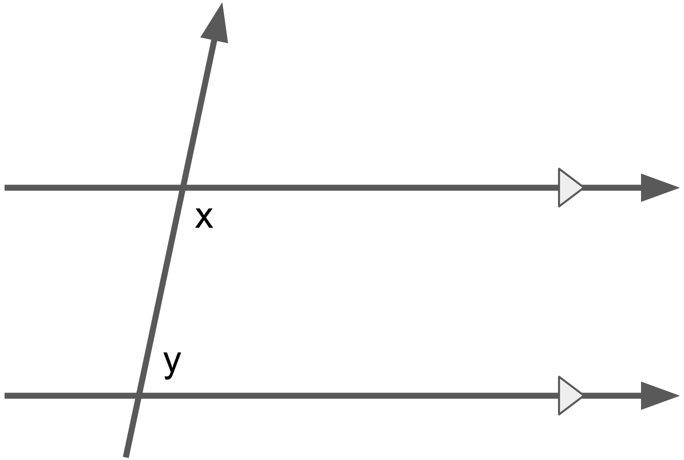
$m\angle$a=

$m\angle$d=

$m\angle$c=

Consecutive Interior Angles Theorem

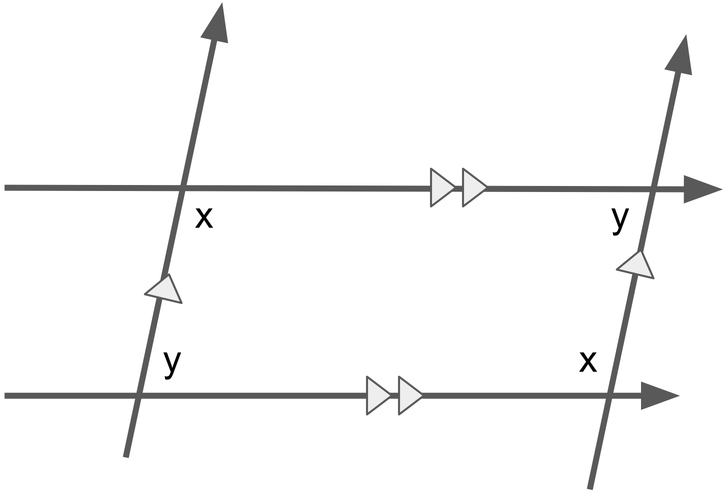
If two parallel lines are cut by a transversal, than the sum of the interior angles will be supplementary.



$m\angle x + m\angle y = 180°$

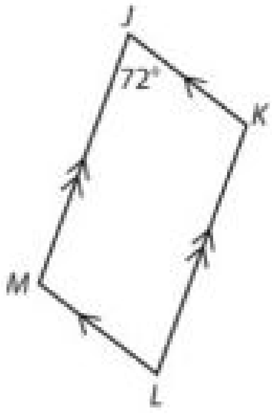
Theorem 8.5

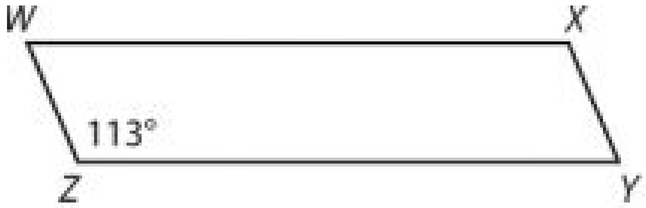
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary



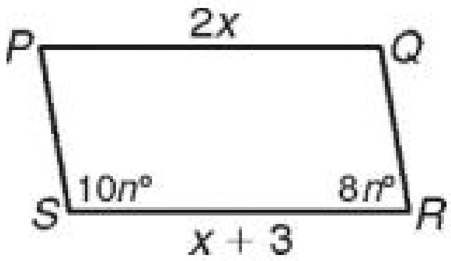
$m\angle x + m\angle y = 180°$

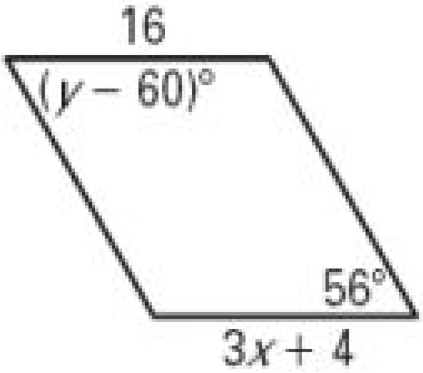
Find the measure of each angle.





Find the missing values.





This is the Dockland office building in Hamburg, Germany.

Each floor is 86 meters long, and the angle of elevation for the acute angle is 66°.

The roof extends over the water 47 meters.

What are the measures of the obtuse angles?

Can we find the length of the sides of the building with this information?

Each floor is 86m long, and the angle of elevation for the acute angle is 66°.

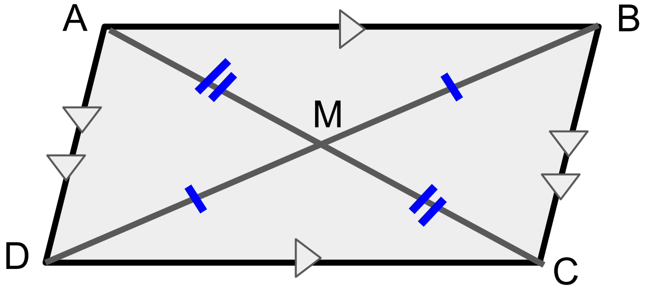
The roof extends over the water 47 meters.

What are the measures of the obtuse angles?

Can we find the length of the sides of the building with this information?

Theorem 8.6

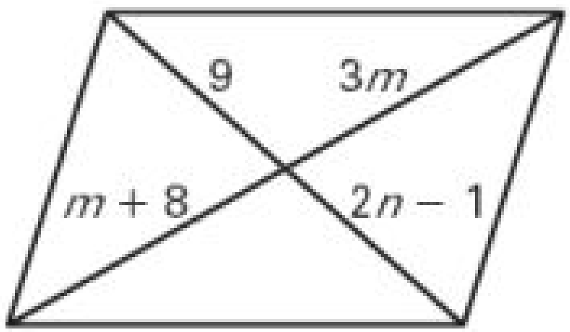
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

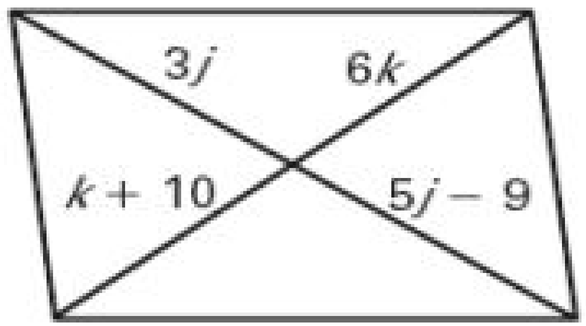


So,

$\overline{AM} \cong \overline{CM}$ and $\overline{DM} \cong \overline{BM}$

Find the values that would make these quadrilaterals parallelograms.





To prove a quadrilateral is a parallelogram:

We use the converse of the theorems we were given, Theorems 8.3 to 8.6.

Theorem 8.3

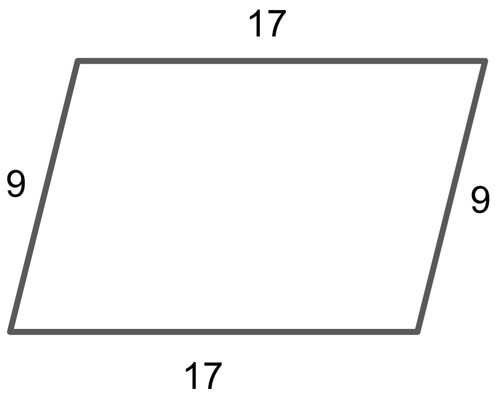
If a quadrilateral is a parallelogram, then its opposite sides are congruent.

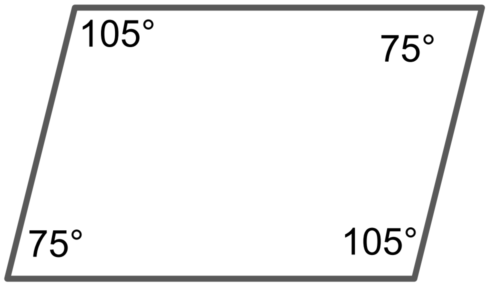
If PQRS is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$

Theorem 8.7 (The converse of Theorem 8.3)

If both pairs of opposite sides of a quadrilateral are congruent, then it's a parallelogram.

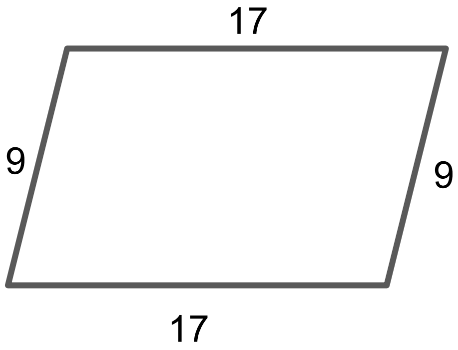
Are these quadrilaterals parallelograms?





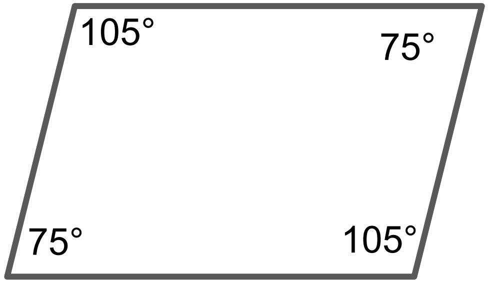
Theorem 8.7 (The converse of Theorem 8.3)

If both pairs of opposite sides of a quadrilateral are congruent, then it’s a parallelogram.



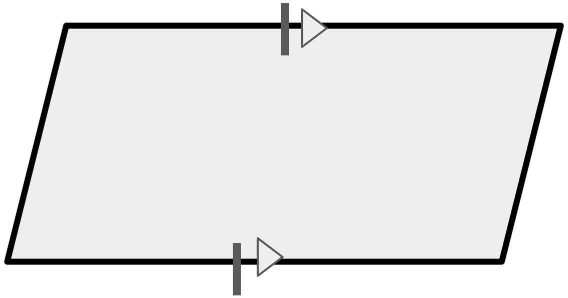
Theorem 8.8 (Converse of Theorem 8.4)

If both pairs of opposite angles of a quadrilateral are congruent, then it is a parallelogram.

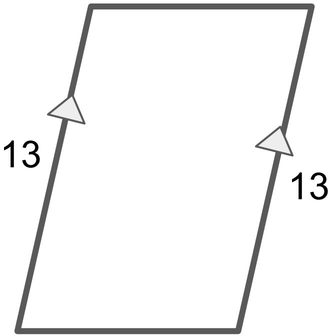


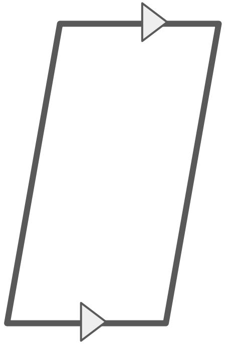
Theorem 8.9

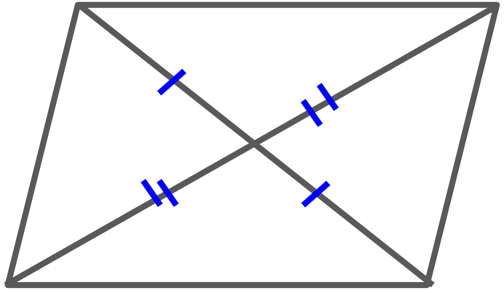
If one pair of opposite sides of a quadrilateral are congruent *and* parallel, then it is a parallelogram

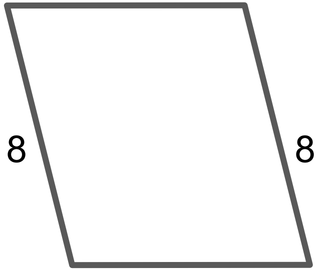


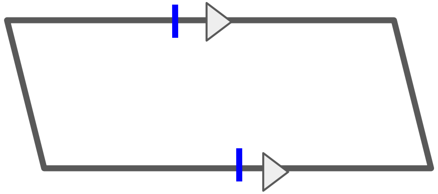
Are these quadrilaterals parallelograms?

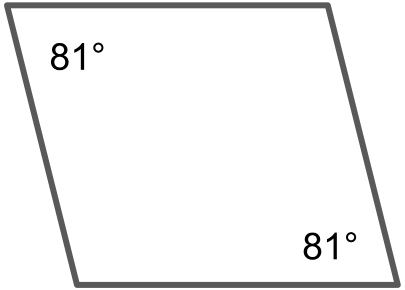






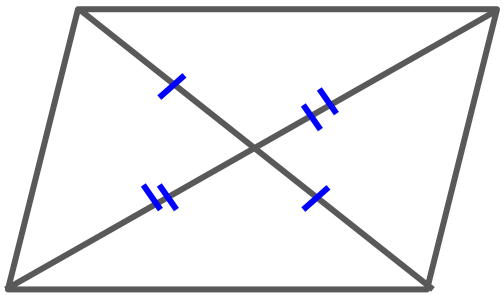






Theorem 8.10 (Converse of Theorem 8.6)

If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.



In Summary:

If a quadrilateral is a parallelogram, then…
it's opposite sides are congruent. *(thm 8.3)*
it's opposite angles are congruent. *(thm 8.4)*
it's consecutive angles are supplementary. *(thm 8.5)*
it's diagonals bisect each other. *(thm 8.6)*

A quadrilateral is a parallelogram if...
both pairs of opposite sides are parallel. *(definition of a parallelogram)*
both pairs of opposite sides are congruent. *(thm 8.7)*
both pairs of opposite angles are congruent. *(thm 8.8)*
one pair of opposite sides are congruent and parallel. *(thm 8.9)*
the diagonals bisect each other. *(thm 8.10)*