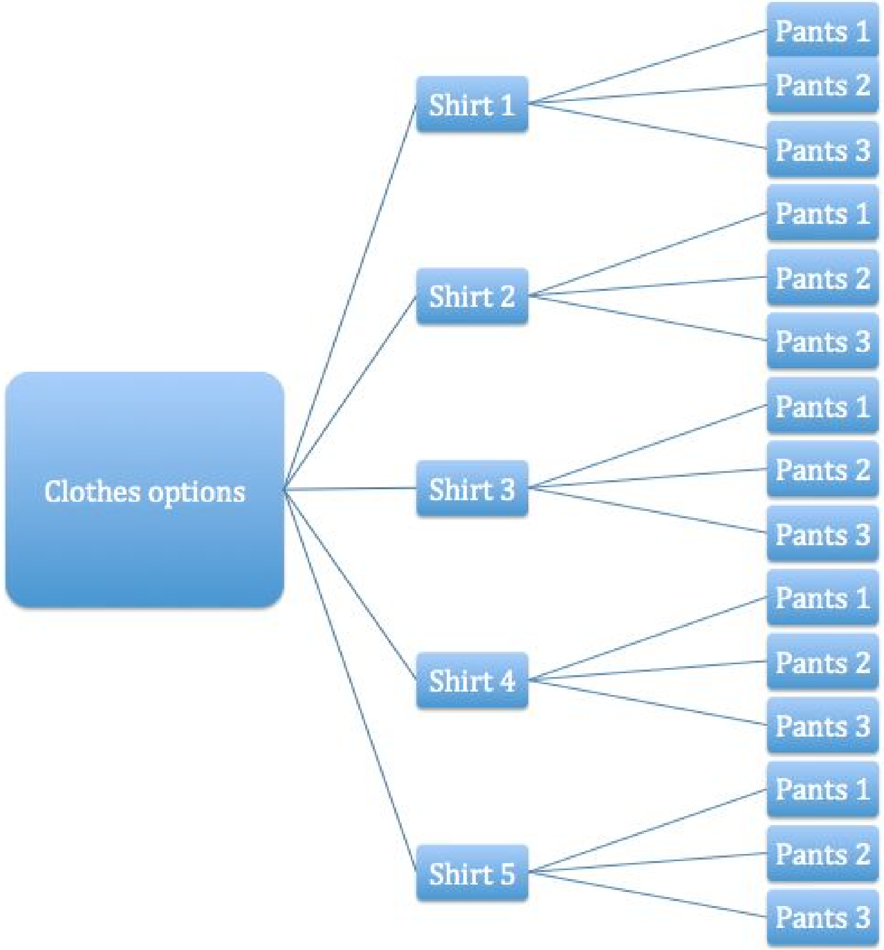
Counting Principles

If you have 5 shirts to choose from and 3 pairs of pants to choose from, how many possible arrangements of clothes do you have?

Tree Diagrams

If you have 5 shirts to choose from and 3 pairs of pants to choose from, how many possible arrangements of clothes do you have?

There are 15 possible ways we could arrange our clothes.



Fundamental Counting Principle

If one event can occur *m* ways, and another can occur in *n* ways, than the number of ways both can occur is *mn***

You can choose 1 of 5 different shirts, so the number of ways our first choice could occur is 5. You can then choose 1 of 3 pairs of pants, so the number of ways our second choice could occur is 3.

So we multiply 5*3 = 15.

There are 15 possible ways we could arrange our clothes.

Fundamental Counting Principle with more than 2 events

If there is more than just two events, or choices, then we multiply the number of ways each of those events could occur.

If we have 5 shirts, 3 pairs of pants, 6 pairs of socks, and 2 pairs of shoes, how many possible arrangement of clothes do we have?

At a sporting goods store, they sell 3 different types of bicycles, each are available in 5 different colors, and 3 different sized wheels. How many bicycle choices does this store offer?

In New York, a license plate on a car has a standard form of 3 letters (A-Z) followed by 3 numbers (0-9).

How many different license plates are possible if the letters and numbers can be repeated?

How many different license plates are possible if the letters and numbers **cannot** be repeated?

Permutations - Order matters

Permutations are a way of counting the number of ways *n* objects can be arranged.

So if we have the letters A, B, and C, we can arrange them as:

ABC, ACB, BAC, BCA, CAB, CBA.

We can use the fundamental theorem of counting here.

3 ways to choose the first letter * 2 ways to choose the second letter * 1 way to choose the last letter.

The number of permutations will be *n!*

There are 5 teams competing in a race. How many ways can the teams finish the race?

Permutations when picking *r* objects

I want to pick a group of 5 students. The order I pick them in will determine the role they play in the group.

In a class of 20, how many different groups can be arranged?

We will use the formula: ${}_nP_r = \frac{n!}{(n-r)!}$

n is the number of distinct objects in a collection

r is the number of objects selected

Permutations with Repetition

If we had the numbers 1, 2, and 1, and we considered 1 and 1 *distinct* or not the same, then we would have 6 permutations or orders that we could arrange them in.

121 121 112 112 211 211

If we consider 1 and 1 the same, then we would only have 3 ways we could arrange the letters that would be different.

112 121 211

We are using the formula:

$$\frac{(number\ of\ objects)!}{(number\ of\ times\ object\ 1\ is\ repeated)!\,(number\ of\ times\ another\ is\ repeated)!\ \ldots}$$

If we have a set of 10 numbers: 1, 2, 1, 3, 5, 2, 8, 5, 7, 5, how many distinguishable ways can they be arranged?

1 is repeated 2 times, 2 is repeated 2 times, 5 is repeated 3 times.

$$\frac{(10)!}{2!\cdot 2!\cdot 3!} = \frac{10!}{6\cdot 2\cdot 2} = \frac{10!}{24} = 151200$$

Given the word MISSISSIPPI. How many distinguishable ways can we arrange the letters?

Combinations - Order doesn’t matter

If I wanted to choose a group of 4 students out of a class of 20, how many ways could I do that?

Combinations

n is the number of objects in a collection.

r is the number of objects being selected from that collection.

$$_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

If I wanted to choose a group of 4 students out of a class of 20, how many ways could I do that?

If I drew 5 random cards out of a 52 card deck, how many different 5-card hands could I draw?

Multiple Events

If there is more than one event occurring, look for keywords.

If we are looking at if event A ***and*** event B happens, we ***multiply*** the combinations.

If we are looking at if event A ***or*** event B happens, we ***add*** the combinations.

Student senate consists of 6 seniors, 5 juniors, 4 sophomores, and 3 freshmen. How many different committees of exactly 2 seniors and 2 juniors can be chosen?

Student senate consists of 6 seniors, 5 juniors, 4 sophomores, and 3 freshmen. How many different committees of exactly 2 seniors and 2 juniors can be chosen?

So, of the 6 seniors, we want to choose exactly 2 ***and*** of the 5 juniors we want to choose exactly 2.

So we can use ${}_6C_2 * {}_5C_2 = 150$.

There are 150 different ways we can have a committee of exactly 2 seniors and 2 juniors.

Student senate consists of 6 seniors, 5 juniors, 4 sophomores, and 3 freshmen. How many different committees of *at most* 4 students be chosen?

In a standard deck of 52 cards, how many possible 5-card hands contain exactly 4 kings and 1 other card?

How many possible 5-card hands contain exactly 5 hearts or 5 diamonds?